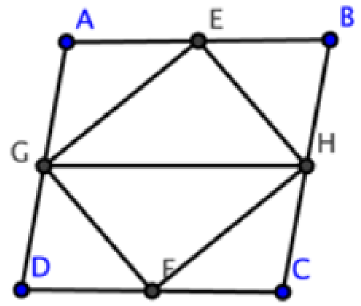
Solutions to short-answer questions

1



a $\triangle GAE \equiv HAF$ (SAS)
$\triangle EBH \equiv FDG$ (SAS)
$\therefore GE = FH$ and $GF = EH$
$\therefore GEHF$ is a parallelogram
$\angle B + \angle A = 180^\circ$ (co-interior angles)
$\angle BEH = (90^\circ - \frac{1}{2}B)$ ($\triangle BEH$ is isosceles)
$\angle AEG = (90^\circ - \frac{1}{2}A)$; ($\triangle AEG$ is isosceles)
$\therefore \angle GAE = 90^\circ$
$\therefore GEHF$ is a rectangle

b 16

2 $(x^2 - y^2)^2 + (2xy)^2 = x^4 - 2x^2y^2 + y^4 + 4x^2y^2$
$= x^4 + 2x^2y^2 + y^4$
$= (x^2 + y^2)^2$
The converse of Pythagoras' theorem gives the result.

3 The diagonals of a rhombus bisect each other at right angles.
Therefore if x cm is the length of each side of the rhombus
$x = \sqrt{9 + 25} = \sqrt{34}$

4 a $x = 7$ cm, $y = 7$ cm, $\alpha = 45^\circ$, $\beta = 40^\circ$

b $\alpha = 125^\circ$, $\beta = 27.5^\circ$

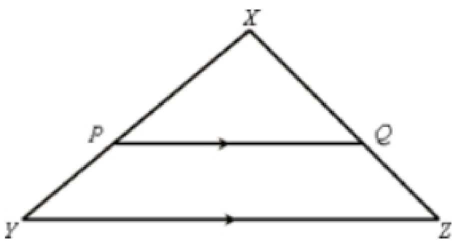
c $\theta = 52^\circ$, $\alpha = 52^\circ$, $\beta = 65^\circ$, $\gamma = 63^\circ$

5 8 m

6 a $\triangle PAQ \equiv \triangle QBO$ (RHS)

b Use Pythagoras' theorem: $\triangle PQR \equiv \triangle ORQ$ (SSS)

7 a



Both triangles share a common angle X.

$\angle XPQ = \angle XYZ$
$\angle XQP = \angle XYZ$

(alternate angles on parallel lines) $\therefore \quad \triangle XPQ \sim \triangle XYZ$ (AAA)

b **a** $\dfrac{XQ}{XZ} = \dfrac{ZP}{XY}$

$\dfrac{XQ}{30} = \dfrac{24}{36} = \dfrac{2}{3}$

$XQ = 20 \text{ cm}$

b $QZ = XZ - XQ$

$QZ = 30 - 20$

$= 10 \text{ cm}$

c $XP : PY = 24 : 12 = 2 : 1$

$PQ : YZ = 2 : 3$

8 **a** $\text{Ratio of areas } ABC : DEF$

$= 12.5 : 4.5$

$= 25 : 9$

$AB : DE = 5 : 3$

$DE = 3 \text{ cm}$

b $AC : DF = 5 : 3$

c $EF : BC = 3 : 5$

9 $\dfrac{h}{21} = \dfrac{1}{2.3}$

$h = \dfrac{2.1}{23} = \dfrac{210}{23} \text{ m}$

10 $BC = 5$ (3- 4- 5 triangle)

So $YB = 2.5$

$\triangle BAC \sim \triangle BYX$

$\dfrac{XY}{YB} = \dfrac{CA}{AB}$

$\dfrac{XY}{2.5} = \dfrac{3}{4}$

$XY = \dfrac{3}{4} \times 2.5 = \dfrac{15}{8}$

11 The triangles are similar (AAA).

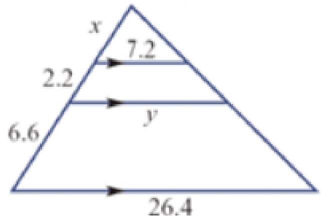
$\dfrac{x-7}{7} = \dfrac{3}{4}$

$4x - 28 = 21$

$4x = 49$

$x = 12.25$

12



If the two sloping lines were extended to form a triangle, then the left side of the top triangle would be given by:

$$\frac{x}{x+8.8} = \frac{7.2}{26.4}$$
$$= \frac{72}{264} = \frac{3}{11}$$
$$11x = 3x + 26.4$$
$$8x = 26.4$$
$$x = 3.3$$

Now compare the top two triangles:

$$\frac{y}{7.2} = \frac{5.5}{3.3} = \frac{5}{3}$$
$$y = \frac{5 \times 7.2}{3}$$
$$= 12$$

13a $\text{Volume of block} = 64\ \text{cm}^3$

$8\ \text{parts} = 64\ \text{cm}^3$

$1\ \text{part} = 8\ \text{cm}^3$

$5\ \text{parts} = 40\ \text{cm}^3$

$3\ \text{parts} = 24\ \text{cm}^3$

Mass of $X = 40 \times \frac{8}{5} = 64\ \text{g}$

Mass of $Y = 24 \times \frac{4}{3} = 32\ \text{g}$

$\text{Total mass} = 96\ \text{g}$

b $X : Y = 64 : 32 = 2 : 1$ (by mass)

c $\text{Volume (cm}^3) : \text{mass (g)}$

$= 64 : 96$
$= 2 : 3$
$= 1000 : 1500$

Volume of $1500\ \text{g}$ block is $1000\ \text{cm}^3$.

d $\sqrt[3]{1000} = 10\ \text{cm} = 100\ \text{mm}$

14a Consider $\triangle BMA$ and $\triangle PAD$.

$$\angle B = \angle P = 90^\circ$$
$$\angle BAM = \angle PDA$$
$$= 90^\circ - \angle PAD$$
$$\angle BMA = \angle PAD$$
$$= 90^\circ - \angle BAM$$
$$\triangle BMA \sim \triangle PAD\ \text{(AAA)}$$

b $BM = 30\ \text{cm}$

$AM = 50\ \text{cm (3--4--5 triangle)}$

Comparing corresponding sides AM and AD:

$AM : AD = 50 : 60 = 5 : 6$

$$\text{Ratio of areas} = 5^2 : 6^2$$
$$= 25 : 36$$

c $\dfrac{PD}{BA} = \dfrac{AD}{MA}$

$\dfrac{PD}{40} = \dfrac{60}{50} = \dfrac{6}{5}$

$PD = \dfrac{6 \times 40}{5} = 48 \text{ cm}$

15a The same units (cm) must be used to compare these quantities.

$200 : 30 = 20 : 3$

b $\dfrac{A}{360} = \dfrac{20^2}{3^2} = \dfrac{400}{9}$

$A = \dfrac{400}{9} \times 360$

$= 16\,000 \text{ cm}^2 = 1.6 \text{ m}^2$

c $\dfrac{V}{1000} = \dfrac{20^3}{3^3} = \dfrac{8000}{27}$

$V = \dfrac{8000}{27} \times 1000$

$= \dfrac{8\,000\,000}{27} \text{ cm}^3$

$= \dfrac{8}{27} \text{ m}^3$

16a Ratio of radii $= 101 : 100 = 1.01 : 1$

Ratio of areas $= 1.01^2 : 1$

$= 1.0201 : 1$

$= 102.01 : 100$

Percentage increase $= 2.01\% \approx 2\%$

b Ratio of volumes $= 1.01^3 : 1$

$= 1.030301 : 1$

$= 103.0301 : 100$

Percentage increase $\approx 3\%$

17a $\dfrac{XY}{BC} = \dfrac{AX}{AB}$

$= \dfrac{3}{9} = \dfrac{1}{3}$

b $\dfrac{AY}{AC} = \dfrac{AX}{AB}$

$= \dfrac{3}{9} = \dfrac{1}{3}$

c $\dfrac{CY}{AC} = \dfrac{2}{3}$

d $\dfrac{YZ}{AD} = \dfrac{CY}{AC}$

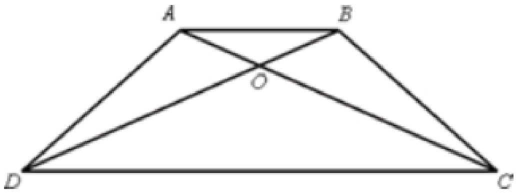
$= \dfrac{2}{3}$

e $\dfrac{\text{area } AXY}{\text{area } ABC} = \dfrac{1^2}{3^2}$

$= \dfrac{1}{9}$

f $$\frac{\text{area } CYZ}{\text{area } ACD} = \frac{2^2}{3^2} = \frac{4}{9}$$

18



Consider $\triangle AOB$ and $\triangle COD$

$\angle AOB = \angle COD$

(vertically opposite angles)

$\angle ABO = \angle CDO$

(alternate angles on parallel lines) $\angle OAB = \angle OCD$

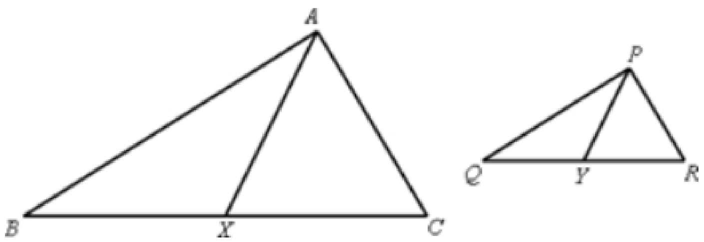
(alternate angles on parallel lines)

$\triangle AOB \sim \triangle COD$ (AAA)

$$\frac{CO}{AO} = \frac{CD}{AB} = \frac{3}{1} = 3$$

$$CO = 3AO$$
$$CO + AO = 4AO$$
$$AC = \$AO$$
$$AO = \frac{1}{4}AC$$

19a



$$\frac{PQ}{AB} = \frac{YQ}{XB}$$

(corresponding sides of similar triangles)

$\angle B = \angle Q$

(corresponding angles of similar triangles)

$\therefore \triangle ABX \sim \triangle PQY$ (PAP)

b $$\frac{AX}{PY} = \frac{AB}{PQ}$$

(similar triangles proven above)

$$\frac{AB}{PQ} = \frac{BC}{QR}$$

(ABC and PQR are similar)

$$\therefore \quad \frac{AX}{PY} = \frac{BC}{QR}$$

Solutions to multiple-choice questions

1 **C** $3x + 66 = 180$

$$3x = 114$$
$$x = 38$$

2 **B** $2x + 270 = 540$

$$2x = 270$$
$$x = 135$$

3 **B**

4 **B** $BC = 10$ by Pythagoras' theorem

Use similar triangles

$\triangle BAD \sim \triangle BCA$

$$\frac{AD}{AB} = \frac{CA}{BC}$$
$$AD = \frac{24}{5}$$

5 **A**

6 **D** $\frac{x}{7} = \frac{3}{5}$

$$x = \frac{3 \times 7}{5}$$
$$= \frac{21}{5}$$

7 **B** $100 \text{ parts} = 400 \text{ kg}$

$$\text{One part} = 4 \text{ kg}$$
$$85 \text{ parts} = 85 \times 4$$
$$= 340 \text{ kg (copper)}$$

8 **D** Cost of one article is $\frac{Q}{P}$.

$$\text{Cost of } R \text{ articles} = \frac{Q}{P} \times R$$
$$= \frac{QR}{P}$$

9 **C** $100 \text{ parts} = 3.2 \text{ m}$

$$1 \text{ part} = \frac{3.2}{100}$$
$$= 0.032 \text{ m} = 3.2 \text{ cm}$$

10 **B** $75 \text{ parts} = 9 \text{ seconds}$

$$1 \text{ part} = \frac{9}{75} = \frac{3}{25} \text{ seconds}$$
$$100 \text{ parts} = \frac{3}{25} \times 100$$
$$= 12 \text{ seconds}$$

11 **D** $10 \text{ parts} = 50$

$$\text{One part} = 5$$
$$\text{Largest part is 6 parts} = 30$$

12 **C** $\text{Ratio of lengths} = 10 : 30 = 1 : 3$

$$\text{Ratio of volumes} = 1^3 : 3^3$$
$$= 1 : 27$$

13 **E** Ratio of lengths $= 4 : 5$

Ratio of volumes $= 4^3 : 5^3$

$= 64 : 125$

14 **E** $\frac{XY}{3} = \frac{12}{10} = \frac{6}{5}$

$$XY = \frac{6 \times 3}{5} = 3.6 \text{ cm}$$

15 **E** $XY' = \frac{2}{3}XY$

Area of triangle $XY'Z'$

$= \frac{4}{9}$ area of triangle XYZ

$= \frac{4}{9} \times 60 = \frac{80}{3} \text{ cm}^2$

Solutions to extended-response questions

1 **a** $\triangle DAC$ and $\triangle EBC$ share a common angle $\angle ACE$ and each has a right angle. Hence $\triangle EBC$ is similar to $\triangle DAC$.

b $\frac{h}{p} = \frac{y}{x+y}$ because corresponding side lengths of similar triangles have the same ratio.

c Using similar triangles $\triangle FAC$ and $\triangle EAB$ (which share a common angle $\angle EAB$ and have a right angle), $\frac{h}{q} = \frac{y}{x+y}$

d $\frac{h}{p} + \frac{h}{q} = h\left(\frac{1}{q} + \frac{1}{q}\right)$ and $\frac{h}{p} + \frac{h}{q} = \frac{y}{x+y} + \frac{x}{x+y}$

$$= \frac{x+y}{x+y} = 1$$

$$\therefore \quad h\left(\frac{1}{p} + \frac{1}{q}\right) = 1$$

e When $p = 4$ and $q = 5$,

$$h\left(\frac{1}{4} + \frac{1}{5}\right) = 1$$

$$\therefore \quad h\left(\frac{5}{20} + \frac{4}{20}\right) = 1$$

$$\therefore \quad \frac{9}{20}h = 1$$

$$\therefore \quad h = \frac{20}{9}$$

2 **a** AF is parallel to BC and AB is parallel to CF

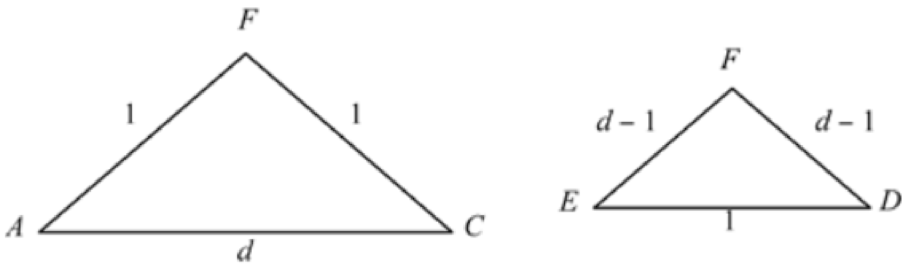
Hence *ABCF* is a rhombus and the length of CF is 1 unit.

b $EF = CE - CF$

$= d - 1$, as required.

c $\triangle ACF$ and $\triangle DEF$ have vertically opposite angles which are equal and they are both isosceles. Hence $\triangle ACF$ and $\triangle DEF$ are similar.

d



$$\frac{d}{1} = \frac{1}{d-1}$$
$$\therefore \quad d(d-1) = 1$$
$$\therefore \quad d^2 - d = 1$$
$$\therefore \quad d^2 - d - 1 = 0$$

e Using the general quadratic formula,

$$d = \frac{1 \pm \sqrt{(-1)^2 - 4 \times 1 \times (-1)}}{2 \times 1}$$
$$= \frac{1 \pm \sqrt{1+4}}{2}$$
$$= \frac{1 \pm \sqrt{5}}{2}$$
$$= \frac{1 + \sqrt{5}}{2}, \text{as } d > 0$$

3 If $DE \parallel AB$ then $\triangle CDE$ is similar to $\triangle ABC$

$$\therefore \quad \frac{CD}{AC} = \frac{CE}{BC}$$
$$\therefore \quad \frac{x-3}{3x - 19 + x - 3} = \frac{4}{x - 4 + 4}$$
$$\therefore \quad \frac{x-3}{4x-22} = \frac{4}{x}$$
$$\therefore \quad x(x-3) = 4(4x-22)$$
$$\therefore \quad x^2 - 3x = 16x - 88$$
$$\therefore \quad x^2 - 19x + 88 = 0$$
$$\therefore \quad (x-11)(x-8) = 0$$
$$\therefore \quad x = 11 \text{ or } 8$$

4 a $\triangle BDR$ and $\triangle CDS$ share a common angle $\angle CDS$ and each has a right angle. Hence $\triangle BDR$ and $\triangle CDS$ are similar.

$\triangle BDT$ and $\triangle BCS$ share a common angle $\angle CBS$ and each has a right angle. Hence $\triangle BDT$ and $\triangle BCS$ are similar.

$\triangle RSB$ and $\triangle DST$ are similar as $\angle RSB = \angle TSD$ (vertically opposite) and $\angle RBS = \angle STD$ (alternate angles).

b
$$\frac{CS}{DT} = \frac{BC}{BD}$$
$$\Rightarrow \frac{z}{y} = \frac{p}{p+q}$$

c
$$\frac{CS}{BR} = \frac{CD}{BD}$$
$$\Rightarrow \frac{z}{x} = \frac{q}{p+q}$$

d $\frac{z}{x}+\frac{z}{y}=z\left(\frac{1}{x}+\frac{1}{y}\right)$ and $\frac{z}{x}+\frac{z}{y}=\frac{p}{p+q}+\frac{p}{p+q}$

$$=\frac{p+q}{p+q}$$

$$=1$$

$$\therefore \quad z\left(\frac{1}{x}+\frac{1}{y}\right)=1$$

$\therefore \quad \frac{1}{x}+\frac{1}{y}=\frac{1}{z}$, as required.

5 a a

$$\frac{QC}{AQ}=\frac{PB}{AP}$$

$$\therefore \quad \frac{6}{2}=\frac{PB}{3}$$

$$\therefore \quad 3\times 3=PB$$

$$\therefore \quad PB=9 \text{ cm}$$

b

$$\frac{PB}{AP}=\frac{BR}{PQ}$$

$$\therefore \quad \frac{9}{3}=\frac{BR}{4}$$

$$\therefore \quad 3\times 4=BR$$

$$BR=12 \text{ cm}$$

c

$$\frac{\text{area } \triangle APQ}{\text{area } \triangle ABC}=\frac{1^2}{4^2}$$

$$=\frac{1}{16}$$

d

$$\frac{\text{area } \triangle BPR}{\text{area } \triangle ABC}=\frac{9^2}{12^2}$$

$$=\frac{81}{144}$$

$$=\frac{9}{16}$$

b i

$$\text{area } \triangle ABC=9\times \text{area } \triangle APQ$$

$$=16a$$

Hence area of $\triangle ABC$ is $16a \text{ cm}^2$.

ii

$$\text{area } \triangle CPQ=\frac{1}{2}(\text{area } \triangle ABC-\text{area } \triangle APQ-\text{area } \triangle BPR)$$

$$=\frac{1}{2}\left(16a-a-\frac{9\times 16a}{16}\right)$$

$$=\frac{1}{2}\times 6a$$

$$=3a$$

Hence area of $\triangle CPQ$ is $3a \text{ cm}^2$.

6 $\dfrac{\text{area } \triangle ADE}{\text{area } \triangle ABC} = \dfrac{1}{9}$

$= \dfrac{1^2}{3^2}$

$\therefore \quad \dfrac{AD}{AB} = \dfrac{AE}{AC}$

$= \dfrac{1}{3}$

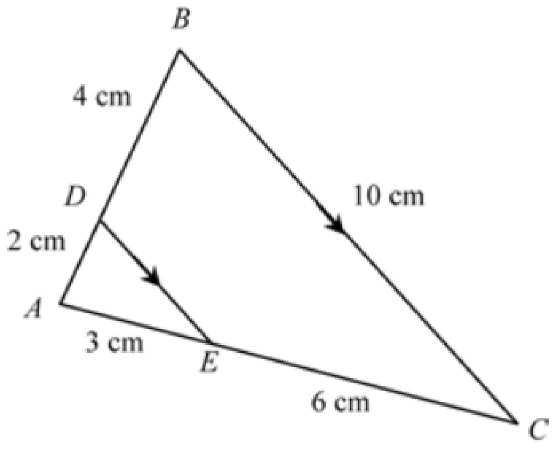
$\therefore \quad AD = \dfrac{1}{3}AB$

$= \dfrac{1}{3} \times 6$

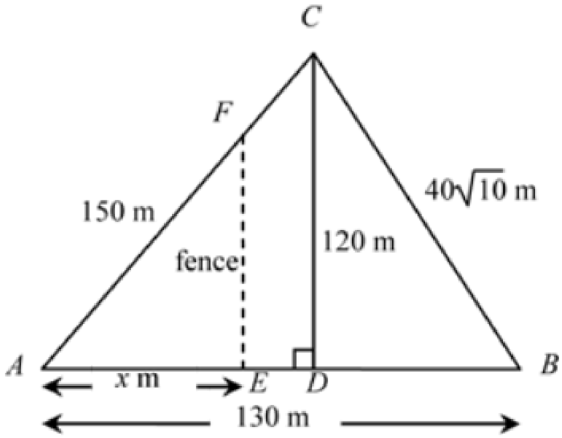
$= 2$

$\therefore \quad AE = \dfrac{1}{3}AC$

$= \dfrac{1}{3} \times 9 = 3$



7 The length of BC should be given as $40\sqrt{10}$ metres.



$\text{area } \triangle AEF = \dfrac{1}{2}\text{area } \triangle ABC$

$= \dfrac{1}{2}\left(\text{area } \triangle ACD + \text{area } \triangle BCD\right)$

$= \dfrac{1}{2}\left(\dfrac{1}{2}\sqrt{150^2 - 120^2}(120) + \dfrac{1}{2}\sqrt{(40\sqrt{10})^2 - 120^2}(120)\right)$

$= \dfrac{1}{2}\left(\dfrac{1}{2}(90)(120) + \dfrac{1}{2}(40)(120)\right)$

$= \dfrac{1}{2}(5400 + 2400)$

$= 3900$

$\dfrac{\text{area } \triangle AEF}{\text{area } \triangle ACD} = \dfrac{3900}{5400}$

$= \dfrac{13}{18}$

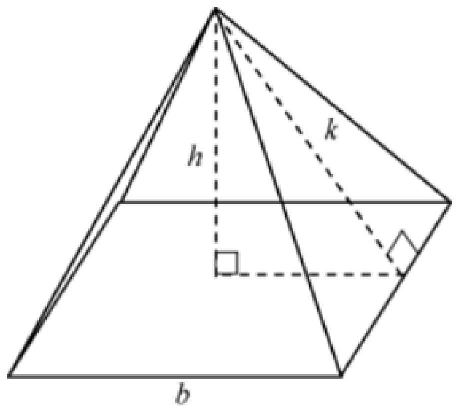
$= \left(\dfrac{\sqrt{13}}{\sqrt{18}}\right)^2$

$\therefore \quad \dfrac{x}{AD} = \dfrac{\sqrt{13}}{\sqrt{18}}$

$\therefore \quad x = \dfrac{\sqrt{13} \times 90}{\sqrt{18}}$

$= 15\sqrt{26} \text{ m}$

8



$$\text{Area of a triangular face} = \frac{1}{2}bk$$

$$h^2 = \frac{1}{2}bk$$

$$h^2 = k^2 - \left(\frac{1}{2}b\right)^2$$

$$= k^2 - \frac{1}{4}b^2$$

$$\therefore \quad k^2 - \frac{1}{4}b^2 = \frac{1}{2}bk$$

$$\therefore \quad 4k^2 - b^2 = 2bk$$

$$\therefore \quad 4k^2 - 2bk - b^2 = 0$$

$$\therefore \quad k = \frac{2b \pm \sqrt{4b^2 + 16b^2}}{8}$$

$$= \frac{2b \pm \sqrt{20b^2}}{8}$$

$$= \frac{b \pm \sqrt{5}b}{4}$$

$$= \frac{b(1+\sqrt{5})}{4} \qquad \text{since } k > 0$$

$$\therefore \quad k = \frac{b}{2}\phi \qquad \text{since } \phi = \frac{1+\sqrt{5}}{2}$$

$$\therefore \quad k : \frac{b}{2} = \phi$$